MAS439/MAS6320 - PROBLEMS - WEEK 1

Question 1 is relatively straightforward, and Question 2 requires deeper understanding. I generally will try to put some harder questions at the very end.

Question 1. *Let M be an R-module. Consider the set*

$$\mathrm{Ann}(M) = \{r \in R : rm = 0 \ \textit{for all} \ m \in M\} \subset R.$$

(a) *Show that* $\mathrm{Ann}(M)$ *is an ideal in R. This ideal is called the annihilator of M. (2 marks)*
(b) *Show that for any ideal $I \subset \mathrm{Ann}(M)$, M has a natural structure of an R/I-module. (2 marks)*

Question 2. (a) *Let k be a field and $R = k[t]$, the polynomial ring. Prove that an R-module is the same thing a k-vector space V together with a linear map $A : V \to V$. That is, you need to prove that an R-module M determines a pair (V, A) and show that conversely the pair (V, A) determines the module M (2 marks).*
(b) *Let $\mathcal{C} = R - mod$ denote the category consisting of of R-modules as objects and R-module homomorphisms as morphisms. Define a category $\mathcal{D}$ whose objects are pairs (V, A) as above with morphisms defined in such a way that there is an equivalence of categories $\mathcal{D} \simeq \mathcal{C}$. (2 marks)*
(c) *Generalize all of the above to the case $R = k[t_1, \ldots, t_n]$ the polynomial ring in n variables. (2 marks)*