

MAS439/MAS6320 - PROBLEMS - WEEK 2

Question 1. Let k be a field and V be a k -vector space. Prove that the following conditions are equivalent:

- (1) V is Noetherian
- (2) V is Artinian
- (3) V is finite dimensional

(3 marks)

Question 2. Let k be a field and let R be the quotient ring $k[x, y]/(y^2)$. Write down the basis of R as k -vector space. Investigate whether R is Noetherian or Artinian as:

- (a) k -vector space
- (b) R -module
- (c) $k[x]$ -module
- (d) $k[y]/(y^2)$ -module

(4 marks)

Question 3. Investigate whether $R = \mathbb{C}[x_1, \dots, x_n]$ ($n \geq 1$) is Noetherian/Artinian ring geometrically, that is using the correspondence between ideals and algebraic sets. (3 marks)