MAS439/MAS6320 - PROBLEMS - WEEK 4

Question 1. *Give an example of a ring R with $J(R) \neq Nil(R)$ (with a proof that your example works). (2 marks)*

Question 2. *Prove parts (b), (c) of Nakayama's Lemma (Theorem 4.9 in the notes). (5 marks)*

Question 3. *Let R be a local Noetherian ring with maximal ideal $m \subset R$. Consider a descending chain*

$$\cdots \subset m^{n+1} \subset m^n \subset \cdots \subset m \subset R$$

of ideals in R. Recall that by definition m^n is generated by products $a_1 \cdots a_n$ of elements $a_i \in m$.

Show that if R is an integral domain and not a field, then the chain above is strictly descending. (3 marks)