MAS439/MAS6320 - PROBLEMS - WEEK 8

Question 1. *Show that Krull's dimension of an Artinian ring is zero. (3 marks)*

Question 2. *Let k be an algebraically closed field.*

(a) *Find the dimension of $V(x^2+y^2+z^2-1) \subset \mathbb{A}^3$ and generalize this to find the dimension of $X = V(f) \subset \mathbb{A}^n$ for $f \in k[x_1, \ldots, x_n]$.*

(b) *Find the dimension of $V(x^2+y^2+z^2-1, xy) \subset \mathbb{A}^3$ and generalize this to find the dimension of $X = V(f, g) \subset \mathbb{A}^n$ for $f, g \in k[x_1, \ldots, x_n]$.*

[Note: there are various cases to consider in general; you can assume $X \neq \emptyset$ (or we can say that the dimension of an empty set is negative infinity).] (4 marks)

Question 3. *Deduce Theorem 5.15 from the Going Up Theorem. (3 marks)*