MAS439/MAS6320 - PROBLEMS - WEEK 9

Question 1. *Let X and Y be algebraic sets. Prove that $X \times Y$ is irreducible if and only if X and Y are irreducible. (3 marks).*

Question 2. (a) *Prove Proposition 9.8 from the notes.*

(b) *Figure out whether statements below are true or false. For each statement, if it is true, give a proof, and if it is false, give a counterexample:*

- (a) *If $f : M_1 \to M_2$ is a surjective R-module homomorphism and N an R-module, then $f \otimes id : M_1 \otimes N \to M_2 \otimes N$ is surjective.*
- (b) *If $f : M_1 \to M_2$ is an injective R-module homomorphism and N an R-module, then $f \otimes id : M_1 \otimes N \to M_2 \otimes N$ is injective.*

[Hint: one of these is false, the other is true.] (3 marks)

Question 3. *Let X, Y, S be algebraic sets with polynomial maps $\phi : X \to S$, $\psi : Y \to S$. This makes $k[X]$, $k[Y]$ into $k[S]$-algebras. Find geometric meaning for $k[X] \otimes_{k[S]} k[Y]$. (4 marks)*