SEMINAR ON THE SINGULARITY CATEGORY
SHEFFIELD SPRING 2017

There are three essentially equivalent ways of assigning a triangulated category to a singular variety $X = \mathrm{Spec}(R)$ [Eisenbud 1980, Buchweitz 1987, Orlov 2004].

- The quotient category $\mathcal{D}^{sg}(X)$ of the category of complexes of finitely generated modules (coherent sheaves) by the category of complexes of projective modules of finite rank (vector bundles)
- The category of $\underline{\mathrm{MCM}}(X)$ maximal Cohen Macaulay modules (coherent sheaves) with projective modules quotiented out
- In the case X appears as a singular fiber of a map $W : Y \to \mathbb{A}^1$, for instance when X is a hypersurface singularity $k[x_1, \ldots, x_n]/(W)$, singularity category is defined as the category of matrix factorizations $\mathrm{MF}(W)$

Main references for singularity categories are on my webpage:
`http://e-shinder.staff.shef.ac.uk/singularity-seminar.htm`.

In the seminar we start by introducing basic properties of singularities (Cohen-Macaulay, Gorenstein and so on), discuss the three categories and equivalences between them, and proceed to topics such as Knörrer periodicity, graded categories of singularities, McKay correspondence, Homological Mirror Symmetry in their relation to the singularity category. At the end we go to more specific topics of interest to participants.

Talks are up to 1 hour and 30 minutes. Meetings take place on Mondays, 2.00-3.30 in J11.

List of talks

1. Feb. 6. **Cohen-Macaulay modules**
   Speaker: Moty
2. Feb. 13. **Maximal Cohen Macaulay modules over hypersurfaces: matrix factorizations and periodic resolutions**
   Speaker: Moty
3. Feb. 20. **Equivalence** $\mathcal{D}^{sg}(X) \simeq \underline{\mathrm{MCM}}(X)$ [Orlov]
   Speaker: Nebojsa
4. Feb. 27. **BGG correspondence** $\mathcal{D}^b(\mathbb{P}(V)) \simeq \mathcal{D}^{sg}(\Lambda^*(V^\vee))$ [Gelfand-Manin, Bernstein-Gelfand-Gelfand, Happel]
   Speaker: Evgeny
   Two weeks break (Moty away in Nanjing)
5. Mar. 20. **Knörrer periodicity** $\mathcal{D}^{sg}(R[x,y]/(f+xy)) \simeq \mathcal{D}^{sg}(R/f)$ [Orlov, Yoshino, Leuschke-Wiegand]
   Speaker: Evgeny
6. Mar. 27. **Graded singularity category** [Orlov]
   Speaker: John G.
   Three weeks Easter break
7. Apr. 24. **Matrix factorizations and Homological Mirror Symmetry** [Orlov]
   Speaker: Sven
8. May 1st *is Bank Holiday - will need to move from Monday.* **McKay correspondence / G-actions / AR-quivers**, or something similar [References: Yoshino, Auslander-Reiten-Smalo]
   Speaker: Joe
9. May 8. **Frobenius twists**
   Speaker: Khaled
10. May 15. **Categorical resolutions**
    Speaker: Barbara